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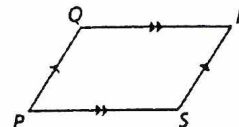
Properties of Parallelograms

Lesson Objective **WBAT** prove and apply theorems about properties of parallelograms.

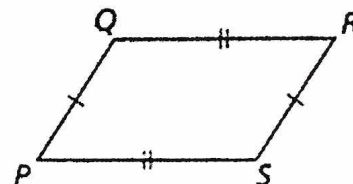
Parallelogram: A quadrilateral with both pairs of opposite sides parallel.

The theorems below describe other properties of parallelograms.

★DEFA★

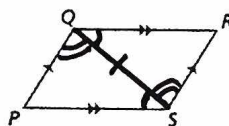
**Parallelogram Opposite Sides Theorem**

If a quadrilateral is a parallelogram, then the opposite sides are congruent.



PROOF: **Given**: PQRS is a parallelogram

Prove: $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$



Statements

Reasons

① PQRS is a ||-gram

② Draw $\overline{QS}$

③ $\overline{QR} \parallel \overline{SP}$, $\overline{PQ} \parallel \overline{SR}$

④ $\angle RQS \cong \angle PSQ$

$\angle RSQ \cong \angle PQS$

⑤ $\overline{QS} \cong \overline{SQ}$

⑥ $\triangle RQS \cong \triangle PSQ$

⑦ $\overline{RQ} \cong \overline{PS}$, $\overline{PQ} \cong \overline{RS}$

① given

② thru 2 pts. is 1 line

③ ||-gram $\rightarrow$ opp. sides ||

④ || $\rightarrow$ alt. int. $\angle$ s $\cong$

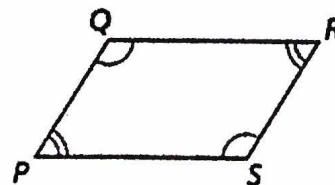
⑤ reflex. prop. $\cong$

⑥ ASA $\cong$

⑦ CPCTC

Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.



PROOF: **Given**: PQRS is a parallelogram

Prove: $\angle P \cong \angle R$ and $\angle Q \cong \angle S$

proof on next page

Statements

Reasons

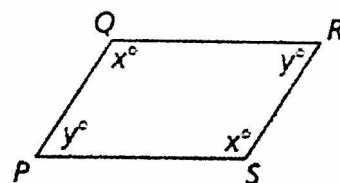
- ① PQRS is a ||-gram
- ② $\overline{QR} \cong \overline{SP}$, $\overline{QP} \cong \overline{SR}$
- ③ Draw $\overline{QS}$
- ④ $\overline{QS} \cong \overline{SQ}$
- ⑤ $\triangle RQS \cong \triangle PSQ$
- ⑥ $\angle R \cong \angle P$
- ⑦ Draw $\overline{PR}$
- ⑧ $\overline{PR} \cong \overline{RP}$
- ⑨ $\triangle QRP \cong \triangle SPR$
- ⑩ $\angle Q \cong \angle S$

- ① given
- ② ||-gram $\rightarrow$ opp. sides $\cong$
- ③ thru 2 pts. is 1 line
- ④ reflex. prop. $\cong$
- ⑤ SSS $\cong$
- ⑥ CPCTC
- ⑦ thru 2 pts is 1 line
- ⑧ reflex. prop. $\cong$
- ⑨ SSS $\cong$
- ⑩ CPCTC

Parallelogram Consecutive Angles Theorem

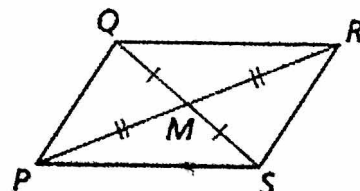
If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

$$x + y = 180$$

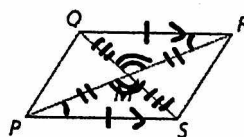


Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.



PROOF: Given: PQRS is a parallelogram
 Diagonals $\overline{PR}$ and $\overline{QS}$ intersect at M
 Prove: M bisects $\overline{PR}$ and $\overline{QS}$



Statements

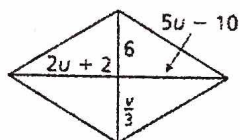
Reasons

- ① PQRS is a ||-gram
Diags. intersect @ M
- ② $\overline{QR} \parallel \overline{SP}$
- ③ $\overline{QR} \cong \overline{SP}$
- ④ $\angle PRQ \cong \angle RPS$
- ⑤ $\angle QMR \cong \angle SMP$
- ⑥ $\triangle RMQ \cong \triangle PMS$
- ⑦ $\overline{MS} \cong \overline{MQ}$, $\overline{MP} \cong \overline{MR}$
- ⑧ M bisects $\overline{PR}$ and $\overline{QS}$

- ① given
- ② ||-gram $\rightarrow$ opp. sides $\parallel$
- ③ ||-gram $\rightarrow$ opp. sides $\cong$
- ④ $\parallel \rightarrow$ alt. int. $\angle$ s $\cong$
- ⑤ vert. $\angle$ s $\cong$
- ⑥ AAS $\cong$
- ⑦ CPCTC
- ⑧ def. of seg. bisector

Example:

1. Find the values of u and v .



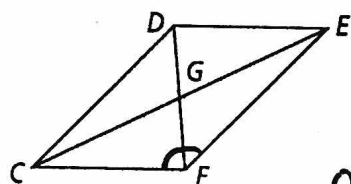
$$2u+2 = 5u-10$$

$$u = 4$$

$$v = \frac{v}{3}$$

$$v = 18$$

2. In parallelogram CDEF, $DE = 74\text{mm}$, $DG = 31\text{mm}$, and $m\angle FCD = 42^\circ$. Answer the following questions and give reasons for each answer.



- a. Find CF

$$74\text{ mm}$$

opp. sides $\cong$

- b. Find $m\angle EFC$

$$138^\circ$$

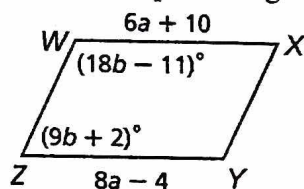
consec.
LS
Supp.

- c. Find DF

$$62\text{ mm}$$

diags
bisect
each other

3. WXYZ is a parallelogram.



- a. Find YZ

$$6a+10 = 8a-4$$

$$a = 7 \quad yz = 52$$

- b. Find $m\angle Z$

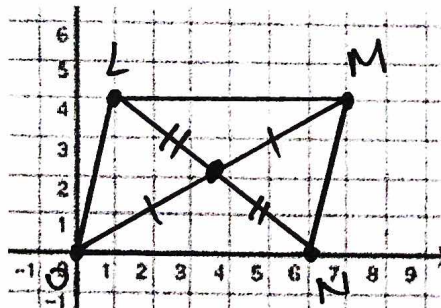
$$18b-11 + 9b+2 = 180$$

$$27b = 189$$

$$b = 7$$

$$m\angle Z = 65^\circ$$

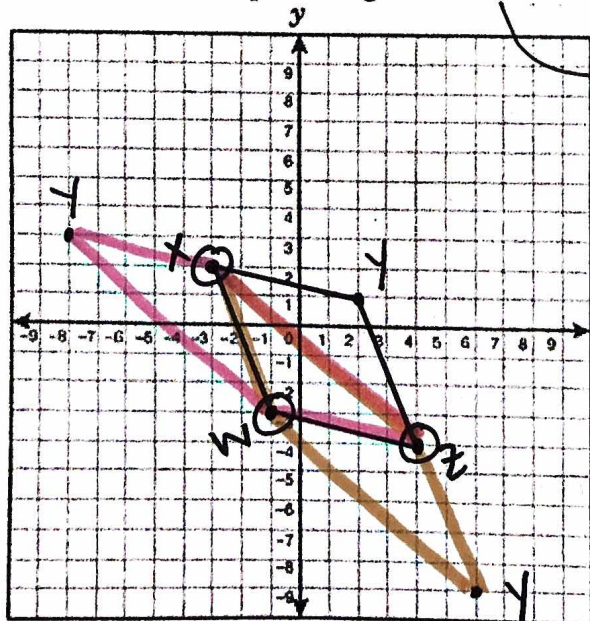
4. Find the coordinates of the intersection of the diagonals of parallelogram LMNO.



*mdpt of either diag.

$$\overline{OM}: \left(\frac{0+7}{2}, \frac{0+4}{2} \right) = (3.5, 2)$$

5. Three vertices of parallelogram WXYZ are shown. Find the coordinates of the fourth vertex, Y.



$$Y(2, 1)$$

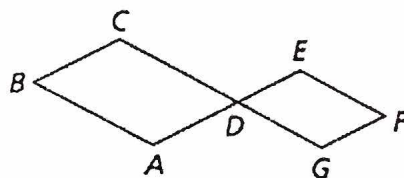
$$ZX \parallel WY: (-8, 3)$$

$$XW \parallel ZY: (6, -9)$$

6. Given: ABCD and GDEF are parallelograms

Prove: $\angle B \cong \angle F$

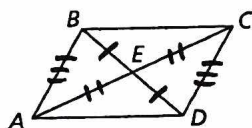
**Remember, since you know that you are given parallelograms, you can use ANY of the theorems we proved already today! ☺ **



Statements	Reasons
① ABCD, GDEF are 11-grams	① given
② $\angle B \cong \angle CDA$, $\angle EDG \cong \angle F$	② 11-gram $\rightarrow$ opp. $\angle$ s $\cong$
③ $\angle CDA \cong \angle EDG$	③ vert. $\angle$ s $\cong$
④ $\angle B \cong \angle EDG$	④ Trans. prop. $\cong$
⑤ $\angle B \cong \angle F$	⑤ Trans. prop. $\cong$

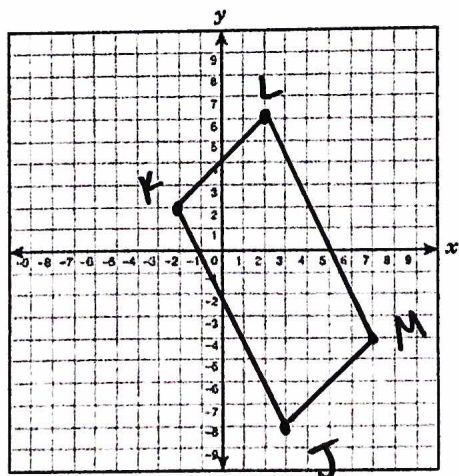
7. Given: ABCD is a parallelogram

Prove: $\triangle AEB \cong \triangle CED$



Statements	Reasons
① ABCD is a 11-gram	① given
② $\overline{AE} \cong \overline{CE}$, $\overline{BE} \cong \overline{DE}$	② 11-gram $\rightarrow$ diags bisect each other
③ $\overline{AB} \cong \overline{CD}$	③ 11-gram $\rightarrow$ opp. sides $\cong$
④ $\triangle AEB \cong \triangle CED$	④ SSS $\cong$

8. Three vertices of parallelogram JKLM are J(3, -8), K(-2, 2), and L(2, 6). Find the coordinates of M.



(7, -4)